Documentation and Validation of EveryCalc's General Mechanism Tool

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May 30, 2020

Abstract

Once upon a time, the man, myth, and legend built the JVN Calc. Since then many people have been devising tweaks and improvements to the system to handle more specific use cases and add additional analysis features. One of the more interesting features is *transient analysis*, also known as *time-to-target* or *sprint-distance* analysis. This considers not only the loads and motors acting on the system, but the accelerated mass, and allows for much more complex analysis of variable loads as well.

1 DC Motor Behavior

To begin with, we're going to consider DC motor behavior.

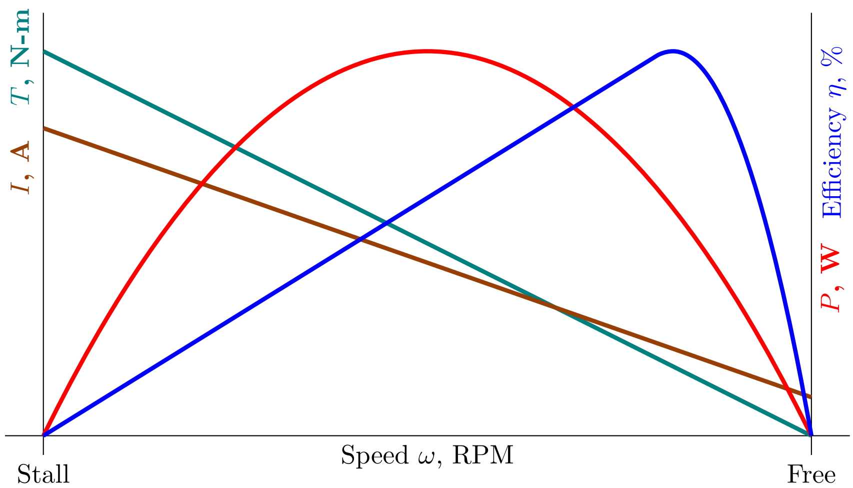


Figure 1: Complete motor curve.

The current and torque, which we are most interested in, could be expressed mathematically as

$$T_m(\omega_m) = T_{m,stall}\frac{\omega_m - \omega_{m,free}}{\omega_{m,free}} \tag{1}$$

$$I_m(T_m) = (I_{m,stall} - I_{m,free})\frac{T_m}{T_{m,stall}} + I_{m,free} \tag{2}$$

2 Gearbox and Coupled Mechanism

Multiple, N motors could be connected to a gear ratio G.

$$T_g(\omega_g) = N\ G\ T_m(G\omega_g)\ \eta_g \tag{3}$$
$$I_g(\omega_g) = N\ I_m(T_g/G) \tag{4}$$

The torque might be limited by a clutch, or by some other friction interface (such as carpet against a wheel). The gearbox also may have non-perfect efficiency, so an efficiency factor can be applied here.

$$T_g(\omega_g) = \min \left\{ \begin{array}{l} N\ G\ T_m(G\omega_g)\eta_g \\ N\mu r \end{array} \right. , \tag{5}$$

where N is the normal force, μ is the friction coefficient, and r is the radius of the friction interface.

Let's consider the following generalized mechanism. Gearbox torque is applied, as is a resistance M_resist. This could be computed from gravity, spring force... or something else entirely.

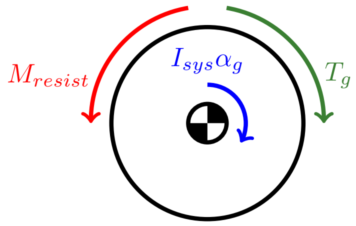


Figure 2: Free-body diagram for the flywheel, with additional resistance M_{resist}.

Using conservation of angular momentum,

$$T_{gbx} - M_{resist} = I_{sys}\alpha_g \tag{6}$$
$$\alpha_g = \frac{d}{dt}\omega_g \tag{7}$$

Solving yields the state equations

$$\frac{d}{dt}\omega_g = \frac{T_g(\omega_g) - M_{resist}}{I_{sys}} \tag{8}$$
$$\frac{d}{dt}\theta_g = \omega_g \tag{9}$$

With initial conditions

$$\omega_g(t=0) = \theta_g(t=0) = 0 \tag{10}$$

This could be adapted to the case of a linear mechanism by substituting

$$M_{resist} = rF_{resist} \tag{11}$$
$$\omega_g = v/r \tag{12}$$
$$\theta_g = x/r \tag{13}$$
$$I_{sys} = mr^2 \tag{14}$$

3 Electrical System and Current Limit

Let's return to our motor model, and pad it out to include the effects of reduced voltage at the motor terminals.

$$T_m = T_{m,stall} \frac{V_m}{V_{nominal}} \frac{\omega_{m,free} \frac{V_m}{V_{nominal}} - \omega_m}{\omega_{m,free} \frac{V_m}{V_{nominal}}} \tag{15}$$

$$I_m = (I_{m,stall} - I_{m,free}) \frac{T_m}{T_{m,stall}} + I_{m,free} \frac{V_m}{V_{nominal}} \tag{16}$$

$$I_m = (I_{m,stall} - I_{m,free}) \frac{V_m}{V_{nominal}} \frac{\omega_{m,free} \frac{V_m}{V_{nominal}} - \omega_m}{\omega_{m,free} \frac{V_m}{V_{nominal}}} + I_{m,free} \frac{V_m}{V_{nominal}} \tag{17}$$

$$\tag{18}$$

Next we consider the voltage on the motor as a result of voltage drop from system resistance R, when ω is the known.

$$V_m = V_{battery} - I_m(\omega_m, \frac{V_m}{V_{nominal}})R \tag{19}$$

$$V_m = V_{battery} - [(I_{stall} - I_{free}) \frac{\omega_{free} \frac{V_m}{V_{nominal}} - \omega_m}{\omega_{free} \frac{V_m}{V_{nominal}}} + I_{free}] \frac{V_m}{V_{nominal}} R \tag{20}$$

$$V_m = \frac{V_{battery} + R(I_{stall} - I_{free}) \frac{\omega_m}{\omega_{free}}}{1 + \frac{R}{V_{nominal} I_{stall}}} \tag{21}$$

The current can then be found from the previous equations

$$I_m = (I_{m,stall} - I_{m,free}) \frac{V_m}{V_{nominal}} \frac{\omega_{m,free} \frac{V_m}{V_{nominal}} - \omega_m}{\omega_{m,free} \frac{V_m}{V_{nominal}}} + I_{m,free} \frac{V_m}{V_{nominal}}$$

If the current found is over the current limit, the current can be set to the current limit, and we can solve backwards.

$$V_m = V_{battery} - RI_m \tag{22}$$

4 Steady-State Calculations

A fair bit of information can be gained from steady-state analysis, so EveryCalc produces these numbers as well. The first is the free speed, given as

$$v_{free} = \frac{\omega_{m,free} r}{G} \tag{23}$$

JVN's speed loss constant (SLC) is used to compute the JVN adjusted speed

$$v_{JVN} = SLC \times v_{free} \tag{24}$$

The running speed can be found at the steady-state condition of $T_g = M_{resist}$. Currently, M_{resist} is evaluated at all parameters being zero, though the use of a numerical method such as bisection could be used to make this a more interesting number.

5 Validation

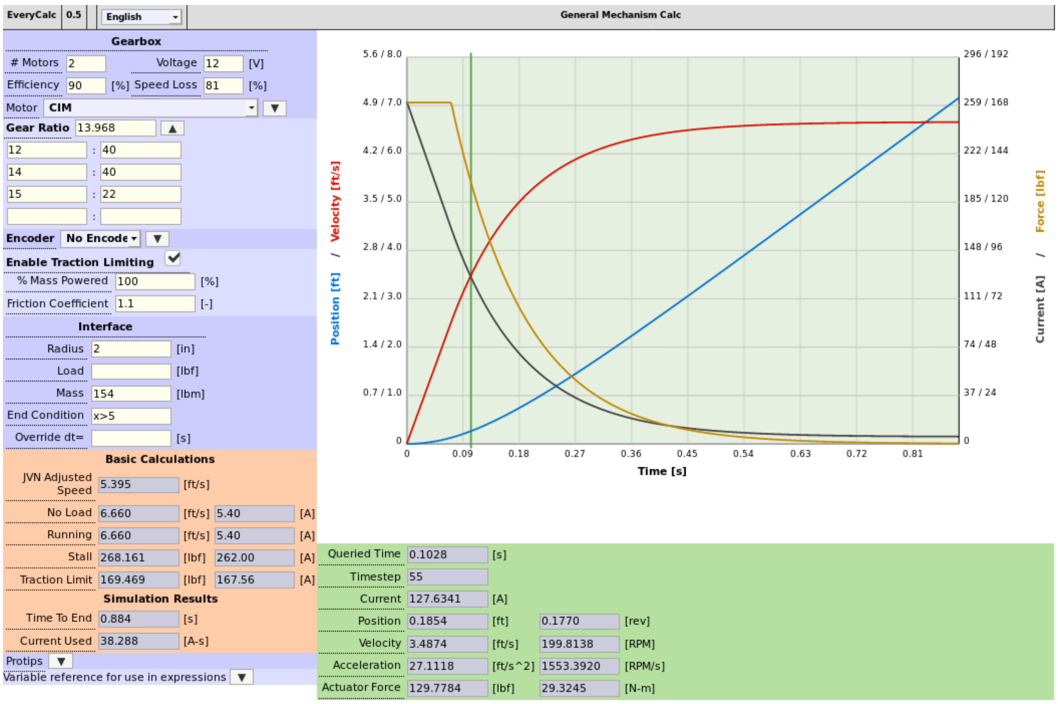
Validation and verification are important aspects in trusting a tools' accuracy. This can be done either to test results or prior art. I would love to obtain test data for sprint distances (and it would be simple to obtain) but currently no such data exists. As such I will compare my results to a couple pre-existing tools.

Case A: Drivetrain

We can compare a drivetrain to the JVN calculator.

1-Speed Drivetrain

	Free Speed (RPM)	Stall Torque (N*m)	Stall Current (Amp)	Free Current (Amp)	Speed Loss Constant	Drivetrain Efficiency
CIM	5330	2.41	131	2.7	81%	90%
# Gearboxes in Drivetrain	# Motors per Gearbox		Total Weight (lbs)	Weight on Driven Wheels	Wheel Dia. (in)	Wheel Coeff
1	2		154	100%	4	1.1
Driving Gear	Driven Gear		Drivetrain Free-Speed	Drivetrain Adjusted Speed	"Pushing" Current Draw per Motor	
12	40		6.66 ft/s	5.39 ft/s	83.75 Amps	
14	40		13.97 : 1	<-- Overall Gear Ratio		
15	22					
1	1					

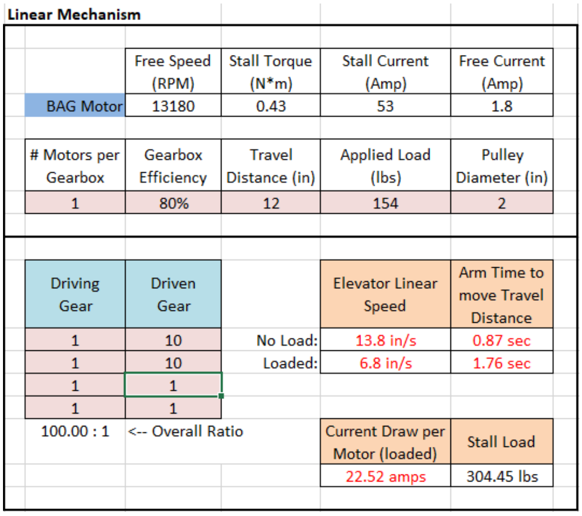


	JVN	EveryCalc V0.5	
Free Speed	6.66	6.669	$ft\ /\ s$
Adjusted Speed	5.39	5.395	$ft\ /\ s$
Current at Traction Limit	$83.75 \times 2 = 167.5$	167.56	A

No issues here other than minor rounding errors.

Case B: Mechanism

We can compare a drivetrain to the JVN calculator.

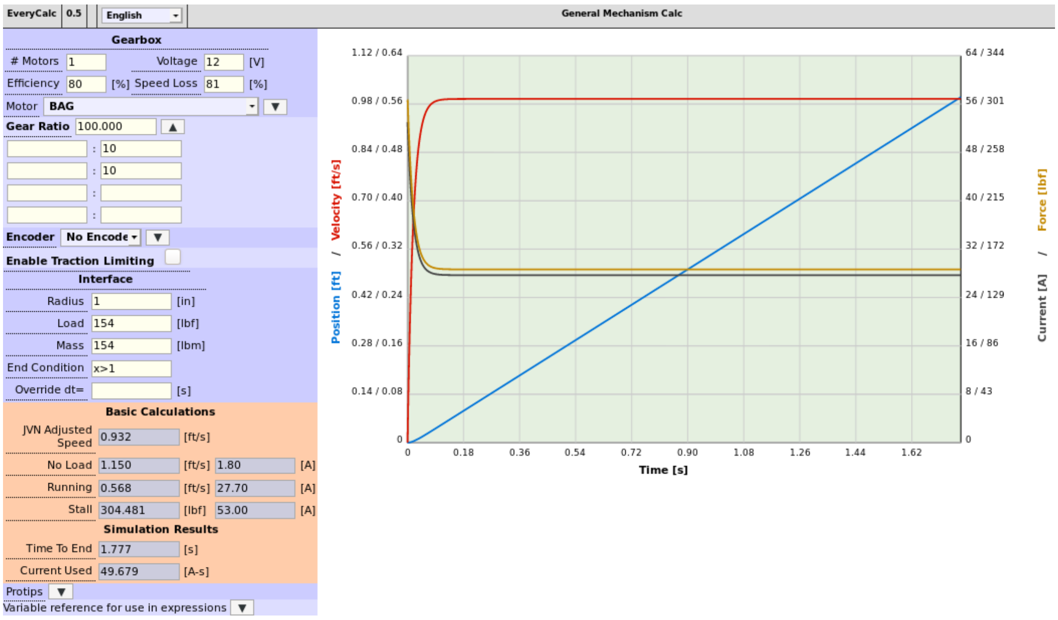


Linear Mechanism

	Free Speed (RPM)	Stall Torque (N*m)	Stall Current (Amp)	Free Current (Amp)
BAG Motor	13180	0.43	53	1.8

# Motors per Gearbox	Gearbox Efficiency	Travel Distance (in)	Applied Load (lbs)	Pulley Diameter (in)
1	80%	12	154	2

Driving Gear	Driven Gear		Elevator Linear Speed	Arm Time to move Travel Distance
1	10	No Load:	13.8 in/s	0.87 sec
1	10	Loaded:	6.8 in/s	1.76 sec
1	1			
1	1			
100.00 : 1	<-- Overall Ratio		Current Draw per Motor (loaded)	Stall Load
			22.52 amps	304.45 lbs



	JVN	EveryCalc V0.5	
Free Speed	1.150	1.150	$ft\ /\ s$
Loaded Speed	0.566	0.568	$ft\ /\ s$
Loaded Current	22.52	27.70	A
Stall Force	304.5	304.481	lbf
Time to Target	1.76	1.777	s

There is a discrepancy in the loaded current number. I believe this is an error in JVN's calculator- his calculation of current does not take into consideration the gearbox efficiency, so is optimistic on current draw.

Note that EveryCalc predicts this mechanism taking slightly longer than JVN does- this is because of the time required to accelerate this mechanism, which the JVN calc does not consider.

Case C: Time-To-Target

We can compare a drivetrain to a modified JVN calculator by apalrd.

Drivetrain Modeling:

In this worksheet, one can graphically model the motion of a FIRST drivetrain (using a numerical analysis method).

Motor Specs (Or Combined Motors Specs) for modeling:

Spec Voltage (V)	Free Speed (RPM)	Stall Torque (N*m)	Stall Current (Amp)	Free Current (Amp)
12	5310	4.86	266	5.4

Drivetrain Physical Constants:

Drive Wheel Dia. (in)	Coeff of Friction	Robot Weight (lbs)	"Lifted" Weight (lbs)	Fractional Weight on Drive	System Interia @ Wheel
5	50	154	0	1	0

High Gear Overall Ratio	Low Gear Overall Ratio	Overall Gearbox Efficiency	Wheel-Floor Efficiency	Static Equilibrium Force (lbs)
0.13333333	0.1333333	90%	1	3

Drivetrain Calcs/Conversions:

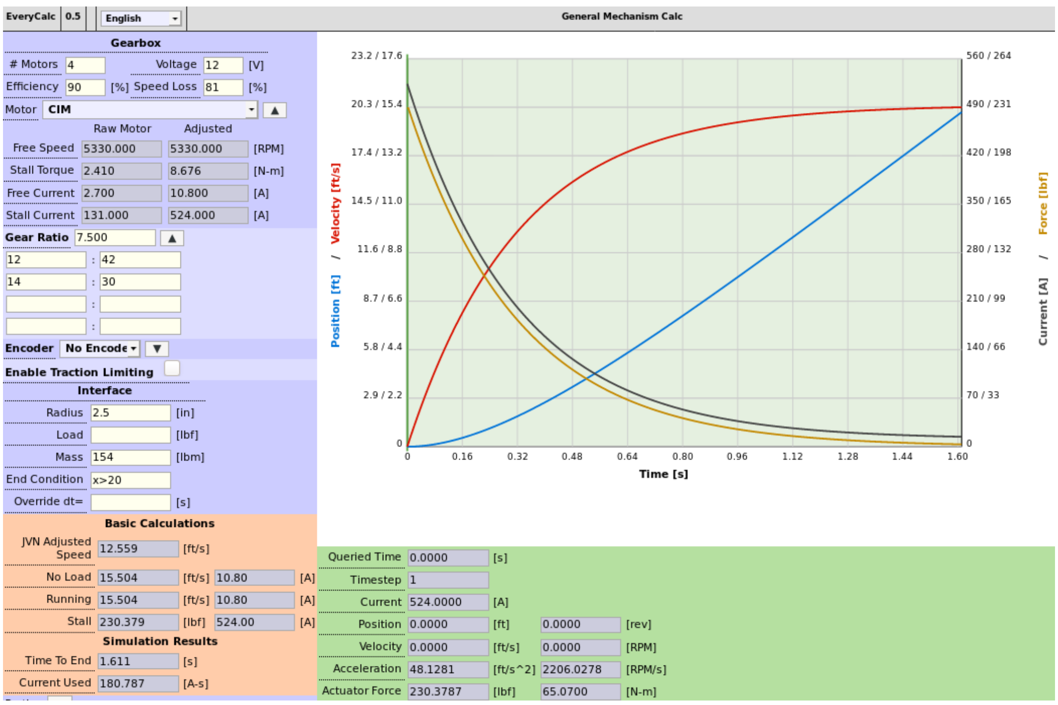
Drive Wheel Rad. (m)	Drive Wheel Circ. (m/rev)	Total Robot Mass (kg)	Static Equilibrium Force (N)	Max Tractive Force (N)
0.0635	0.3989823	69.872958	13.34466	34272.686

Modeling Constants (adjust to change graphs):

Time Step (sec) [.01 is max]	Battery Voltage	Battery Resistance	Battery Voltage Filter Gain
0.01	12	0	0

Reference Speed (ft/s)	Reference Current (Amp)	Reference Distance (ft)	Reference Motor Load (N*m)	Reference Time (sec)	Reference Battery Voltage (v)	HG Time to reach Ref Distance (s)
19.1	100	20	3.5	0.5	12	1.62

Initial Low Gear Speed (ft/s)	Initial High Gear Speed (ft/s)	Initial LG Time (sec)	Initial HG Time (sec)	Initial LG Position (ft)	Initial HG Position (ft)	LG Time to reach Ref Distance (s)
0	0	0	0	0	0	1.62



	JVN/apalrd	EveryCalc V0.5	
Time to Target	1.62	1.611	s

Effectively the same result. Note: the modified calculator with sprint distance does not consider wheel slip due to friction, hence traction limiting is disabled in everycalc for a fair comparison.